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Question Paper Code : 72061

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

First Semester

Civil Engineering

MA 6151 — MATHEMATICS — I

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/ Agriculture Engineering/Automobile Engineering/Biomedical Engineering/ Computer Science and Engineering/Electrical and Electronics Engineering/Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Environmental Engineering/Geoinformatics Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Materials Science and Engineering/Mechanical Engineering/ Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics Engineering/Metallurgical Engineering/Petrochemical Engineering/ Production Engineering/Robotics and Automation Engineering/ Biotechnology/ Chemical Engineering/Chemical and Electrochemical Engineering/Fashion Technology/Food Technology/Handloom & Textile Technology/Industrial Biotechnology/Information Technology/Leather Technology/Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology/Plastic Technology/Polymer Technology/Rubber and Plastics Technology/Textile Chemistry/Textile Technology/Textile Technology (Fashion Technology)/ Textile Technology (Textile Chemistry))

(Regulations 2013)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Two eigenvalues of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 0. What is the third eigenvalue? What is the product of the eigenvalues of A?

2. Find the constants a and b such that the matrix $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3 and –2 as its eigenvalues.

3. Test the convergence of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....\infty$.

4. Examine the convergence of the sequence $u_n = 2n$.

5. Define Evolute and Involute.

6. Find the envelop of the family of lines $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$, θ being the parameter.

7. If $u=\sin^{-1}\left[\frac{x^3-y^2}{x+y}\right]$, then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\tan u$.

8. Find $\frac{du}{dt}$, if $u=\frac{x}{y}$, where $x=e^t$, $y=\log t$.

9. Evaluate : $\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta$.

10. Evaluate : $\int_1^3\int_3^4\int_1^4 xyz\,dxdydz$.

PART B — (5 × 16 = 80 marks)

11. (a) Verify Cayley-Hamilton theorem find A^4 and A^{-1} when $A=\begin{bmatrix}2 & -1 & 2\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$.

Or

(b) Reduce the matrix $\begin{bmatrix}10 & -2 & -5\\ -2 & 2 & 3\\ -5 & 3 & 5\end{bmatrix}$ to diagonal form. (16)

12. (a) (i) Test the convergence and absolute convergence of the series. (8)

$$\frac{1}{\sqrt{2}+1}-\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{4}+1}-\frac{1}{\sqrt{5}+1}+\ldots.$$

(ii) Test for convergence of the series $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^2+1}$. (8)

Or

(b) (i) Test the series $\sum_{n=1}^{\infty}\left(\sqrt{n^2+1}-n\right)$. (8)

(ii) Test the convergence of the sum

$$\frac{1}{1.3}+\frac{2}{3.5}+\frac{3}{5.7}+\frac{4}{7.9}+\ldots$$ (8)

13. (a) (i) Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, considering it as the envelope of its normals. (8)

(ii) Find the envelope of $\frac{x}{a}+\frac{y}{b}=1$, where a and b are connected by $a^2+b^2=c^2$, c being a constant. (8)

Or

(b) (i) Prove that the radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$ is $4a\cos\frac{\theta}{2}$. (8)

(ii) Find the circle of curvature at $(3,4)$ on $xy=12$. (8)

14. (a) (i) A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires the least material for its construction. (8)

(ii) Find the minimum values of x^2yz^3 subject to the condition $2x+y+3z=a$. (8)

Or

(b) (i) Obtain the Taylor series of $x^3+y^3+xy^2$ at $(1,2)$. (8)

(ii) If $u=\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{-1}{2}\cot u$. (8)

15. (a) (i) Change the order of integration and hence evaluate it $\int_{0}^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\,dy\,dx$. (8)

(ii) Evaluate : $\int_0^a\int_0^b\int_0^c (x^2+y^2+z^2)\,dxdydz$. (8)

Or

(b) (i) Evaluate $\iint (x-y)\,dxdy$ over the region between the line $y = x$ and the parabola $y = x^2$. (8)

(ii) Find the value of $\iiint xyz\,dxdydz$ through the positive spherical octant for which $x^2+y^2+z^2 \le a^2$. (8)